

SUBJECT: MATHEMATICS**Holiday Package Class: S5 MPG, MCB & PCM****SECTION A Subjective type questions**

- 1) Earthquake magnitude M and intensity I are related by $\log I = 1.5M + 2$.
- (a) Evaluate intensity for $M=6$. (b) Determine magnitude if intensity $I=1000$.
(c) Assess whether a magnitude 7 earthquake is twice as intense as magnitude 6.
- 2) (a) Construct an exponential growth model for a bacterial culture that doubles every 4 hours.
(b) Create a logarithmic decay model for a radioactive isotope with half-life 10 days.
(c) Design an exponential model for savings in a bank with continuous interest rate 5% per year. (d) Suppose that the amount of money in a bank is given by $P(t) = 500 + 100\sin(t)$ where t is in years. During the first 5 years in which the account is opened when is the amount of money in the account decreasing?
- 3) (a) Create a trigonometric equation whose solution is $x = 90^\circ$.
(b) Construct a simple inequality involving $\sin x$ valid in the first quadrant.
(c) Construct an equation of displacement for SHM with amplitude 5 cm.
(d) Using Snell's law, assess whether refraction occurs when light is incident normally on glass.
(e) Create and solve a trigonometric inequality whose solution lies strictly in the second quadrant.
- 4) (a) State the general solution of $\sin x = \sin \alpha$. (b) Find the domain of $\sqrt{\sin x}$ (c) Find the 10th term of the sequence defined by $a_1 = 3$ and $a_{n+1} = 2a_n - 1$.
(d) Find the sum of the first 12 terms of the arithmetic sequence whose 3rd and 7th terms are 10 and 22 respectively. (e) Explain the effect of squaring both sides of an equation
(f) If the zeros of $f(x)$ are 3 and -4, What are the zeros of the function $g(x) = f\left(\frac{x}{3}\right)$?
- 5) a) Write the formula for linear interpolation
b) Describe the principle of locating a root using sign change.
c) Check whether displacement and acceleration are in the same direction in SHM

- d) A student claims the solution of $\tan 2x = \tan x$ is $x = k\pi$. Evaluate the claim
- e) Evaluate $\tan(\arctan 3 + \arctan 2)$ without using a calculator
- f) Find the probability of obtaining at least 2 heads when a coin is tossed thrice.
- 6) Given $f(x) = x^3 - x - 1$.
- Show that a root lies between $x = 1$ and $x = 2$.
 - Find the first approximation using bisection method.
 - Find the second approximation.
 - Write the Newton–Raphson iteration formula.
 - Take initial guess $x_0 = 1.5$ and find x_1 using Newton–Raphson iteration.
- 7) i) Solve the equation $f^{-1}(x) = gf(x)$ where $f: x \mapsto 3x + 2$; $g: x \mapsto 4x - 12$
- ii) Perform 3 iterations It is given that a is a positive constant.
- Sketch on a single diagram the graphs of $y = |2x - 3a|$ and $y = |2x + 4a|$
 - State the coordinates of each of the points where each graph meets an axis.
 - Solve the inequality $|2x - 3a| < |2x + 4a|$
- of the bisection method to estimate the value of $\sqrt[3]{7}$ starting with $x_0 = 1.5$
- iii) If $\sin 36^\circ \cos 12^\circ = p$ and $\cos 36^\circ \sin 12^\circ = q$, determine an expression in terms of p and q for
- $\sin 48^\circ$
 - $\cos 48^\circ$
- iv) Three numbers, A, B, C in that order, are in geometric progression with common ratio r . Given further that $A, 2B, C$ in that order are in arithmetic progression, determine the possible values of r .
- 8) Solve $AX - 2M = 0$ if $A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ and $M = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$
- 9) The following table gives a number of advertisement (x_i) and the sales in hundreds of dollars (y_i) of a certain sport company.

(x_i)	1	2	3	4	5	6
(y_i)	41	50	54	54	57	63

- Find the standard deviation for (x_i) and (y_i)
- Calculate the correlation coefficient r
- Find the equation of regression line Y on X
- For 7 number of advertisements. Estimate the volume of sales

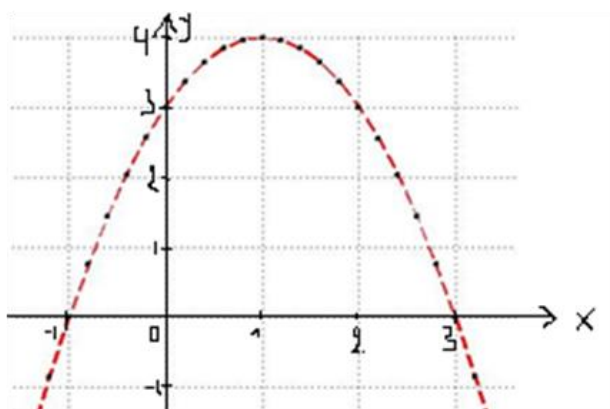
- 10) The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying joint probability table gives the proportions of individuals in the various ethnic group/blood group combinations

Ethnic group	Blood group			
	O	A	B	AB
1	0.082	0.106	0.008	0.004
2	0.135	0.141	0.018	0.006
3	0.215	0.200	0.065	0.020

Suppose that an individual is randomly selected from the population, and define the events:

$A = \{\text{Blood Type A selected}\}$ $B = \{\text{Blood Type B selected}\}$ $C = \{\text{Ethnic Group 3 selected}\}$

- Calculate $P(A)$, $P(C)$, and $P(A \cap B)$
 - Calculate both $P(A | C)$ and $P(C | A)$, and explain in context what each of these probabilities represent.
 - If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?
- 11) Consider the function $f : x \mapsto x^2 + ax + b$, where a and b are constants.
- It is given that $a = 6$ and $b = -8$. Find the range of f .
 - It is given instead that $a = 5$ and that the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k .
 - Show that if the equation $f(x + a) = a$ has no real roots then $a^2 < 4(b - a)$.
- 12) a) Show that $\frac{\log_a \sqrt{64} + \log_a \sqrt{27} - \log_a \sqrt{125}}{\log_a 12 - \log_a 5} = \frac{3}{2}$ where a is a positive constant
- b) Express $1 - 5t$ as linear combination of $t + 2$, $t - 2t^2$, $3t + 1 - t^2$
- c) Let $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (i) Find M^{-1} (ii) Find α and β so that $(\alpha I + \beta M) = M$
- 13) Study the graph below and answer the following questions



- Determine the coordinates of
 - The vertex (pink)
 - Y_intercept
 - x_intercepts
 - write the equation of the axis of symmetry
 - find the maximum values and the maximum point
- Solve
 - $f(x) = 0$
 - $f(x) > 3$
 - $y = 3$

Section B: Objective type questions

14) An accelerometer measures $x(t) = 0.08\cos(12t)$ m. Find the maximum speed.

A: 0.08 m/s **B:** 0.96 m/s **C:** 1.0 m/s **D:** 12.0 m/s

15) State whether each of the following is TRUE or FALSE

- a) The general solution of $\sin x = \sin \alpha$ is $x = \alpha + 2k\pi$ or $x = \pi - \alpha + 2k\pi$
- b) The identity $1 + \tan^2 x = \sec^2 x$ holds for all real x where defined.
- c) If light travels from air to glass, it bends away from the normal.
- d) Snell's law is given by $n_1 \sin r = n_2 \sin i$

16) State whether each statement is **True (T)** or **False (F)**

- a) The exponential population growth is $P(t) = P_0 e^{kt}$, where $k > 0$.
- b) In population decay, the growth constant k is negative.
- c) The formula $P(t) = P_0 e^{kt}$ applies only to population growth.
- d) The compound interest formula is $A = P \left(1 + \frac{r}{n}\right)^{rt}$.

17) If the sum of the first n terms of a sequence is $S_n = 3n^2 + 5n$ then the n^{th} term is: A. $6n + 5$ B. $6n + 2$ C. $3n + 5$ D. $6n - 3$

18) The population growth is $P(t) = P_0 e^{kt}$. State whether each statement is **True** or **False**

- a) If $k = 0$, the population remains constant for all t .
- b) A population doubling time depends only on the initial population.
- d) Decay models approach zero but never reach zero

19) Solve the equation $\sin 2x = \sin x$, $0 \leq x < 2\pi$.

A. $x = 0, \pi$ B. $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ C. $x = \frac{\pi}{2}, \frac{3\pi}{2}$ D. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

20) Which of the following represents the correct domain of the function $f(x) = \sec x$ A. $\mathbb{R} -$

$\{n\pi: n \in \mathbb{Z}\}$ B. $\mathbb{R} - \left\{(2n+1)\frac{\pi}{2}: n \in \mathbb{Z}\right\}$ c) $[-1,1]$ D. $\mathbb{R}$

21) Match each item in Column A with the correct letter in Column B.

Column A	Column B
1. Interpolation	A. Estimates values outside known data range
2. Bisection method	B. Estimates values within known data range
3. Extrapolation	C. Uses two points to estimate a value
4. Linear interpolation	D. Uses tangent line for root approximation
5. Newton-Raphson method	E. Reduces interval by half each iteration

22) A student claims that the inequality $\sin x > \cos x$ holds for all $x \in (0, \pi)$. Which evaluation is correct?

- A. True, because sine is always positive B. True only in the first quadrant
C. False, because it fails near $x = 0$ D. True for all real values

23) Determine the value of the expression $\sin(\arccos \frac{3}{5})$ A. $\frac{3}{5}$ B. $\frac{4}{5}$ C. $\frac{3}{4}$ D. $\frac{5}{4}$

24) What is the domain of the function $f(x) = 3 \sin^{-1}(3x - 1)$

- A. $[0, \frac{2}{3}]$ B. $[-1, 1]$ C. $\mathbb{R}$ D. $[-\frac{2}{3}, 0]$ E. $[0, 2]$

25) The value of k for which vector $\vec{w} = (1, -2, k)$ of $\mathbb{R}^3$ is a linear combination of vectors $\vec{u}_1 = (3, 0, -2)$ and $\vec{u}_2 = (2, -1, -5)$ is

- A. 8 B. 3 C. $\frac{8}{3}$ D. -8 E. neither

26) If $\lim_{h \rightarrow 0} \left[\frac{\arcsin(a+h) - \arcsin(a)}{h} \right] = \sqrt{2}$,

which of the following could be the value of a ?

- A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\sqrt{3}$ D. $\frac{\sqrt{2}}{2}$ E. 2

27) Work out and circle the letter related to the best answer: Consider three vectors in Euclidian space $\mathbb{R}^3$: $u = -3i + 4j + 12k$, $v = 4i - 3j + 2k$ and $w = mi + 3j - nk$

i) The values of m and n if $u = v \times w$ are equal to

$$A. \begin{cases} m = 0 \\ n = 1 \end{cases} \quad B. \begin{cases} m = 1 \\ n = 0 \end{cases} \quad C. \begin{cases} m = 4 \\ n = 12 \end{cases} \quad D. \begin{cases} m = 12 \\ n = 4 \end{cases}$$

ii) The area of the triangle (O, v, w) is A. 4.5 B. 6.5 C. 9 D. 3

28) i) When the correlation coefficient is zero, the regression lines

- A. coincide B. are parallel to axes
C. are perpendicular D. become the same line E. cannot be drawn

ii) Consider the regression line of Y on X: $Y = 5 + 2X$, with standard deviations $s_X = 4, s_Y = 8$.

(a) The slope b_1 is A. 1 B. 2 C. 4 D. 8 E. 0

(b) The coefficient of correlation r is: A. 0.25 B. 0.5 C. 0.75 D. 1

iii) If regression coefficients are $b_{yx} = 0.5$ and $b_{xy} = 0.8$, then A. $r = 0.65$
B. $r > 1$ C. $r = 0$ D. $r = \sqrt{(0.5 \times 0.8)}$ E. $r = \frac{1}{2}(0.5 + 0.8)$

29) Match Column A with the correct principal value range in Column B

Column A	Column B
1. $\sin^{-1} x$	A. $[0, \pi]$
2. $\cos^{-1} x$	B. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
3. $\tan^{-1} x$	C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
4. $\sec^{-1} x$	D. $[0, \pi], x \neq \pi$
5. $\cot^{-1} x$	E. $(0, \pi)$

30) The bisection method is applied to solve $f(x) = x^2 - 2$. If the interval is $[1, 2]$, what is the first approximated solution? a) 1.25 b) 1.5 c) 1.75 d) 2

31) Using Newton-Raphson, solve one iteration for $f(x) = x^2 - 2$ starting at $x_0 = 1.5$.

a) 1.41 b) 1.45 c) 1.42 d) 1.47

32) A Ferris wheel of radius 20 m rotates once every 5 minutes. The height $h(t)$ of a seat above ground can be modeled by a sinusoidal function. What is the **maximum height**?

- A. 20 m B. 40 m C. 25 m D. 45 m

33) A function f is increasing if $f'(x) > 0$, and decreasing if $f'(x) < 0$

i. Determine the intervals where $f(x) = x^3 - 3x^2 + 2$ is decreasing.

- a) $(-\infty, 0) \cup (2, \infty)$ b) $(-\infty, 1) \cup (2, \infty)$ c) $(-\infty, 2) \cup (3, \infty)$ d) $(-\infty, 0) \cup (3, \infty)$

ii. A tide model is $H(t) = 2 + 1.5 \cos(\frac{\pi t}{6})$ meters, t in hours. Time for high tide is:

- a) 3 hours b) 4 hours c) 6 hours d) 12 hours

34) A radioactive substance decays with half-life of 10 years. If the initial mass is 200 g, how much remains after 30 years? a) 25 g b) 50 g c) 100 g d) 200 g

35) Earthquake A has magnitude 6, Earthquake B has magnitude 8. How many times more intense is B compared to A? a) 10 b) 100 c) 1000 d) 10,000

36) A student sees a bird on top of a 12m high light pole. The student is standing 20m from the base of the pole. At what angle must the student incline her camera to take a picture of the bird? a) 37° b) 31° c) 59° d) 87° e) none of these

37) Find x if $\ln 2$, $\ln(2^x - 1)$ and $\ln(2^x + 3)$ are consecutive terms of an arithmetic progression
a) $\ln 5$ b) 2^x c) $\log_2 5$ d) $\ln(2^x + 1)$ e) None of these

38) A box has 5 red and 3 blue balls. One is chosen at random. Probability it is blue?

- a) $\frac{3}{5}$ b) $\frac{5}{8}$ c) $\frac{3}{8}$ d) $\frac{1}{3}$

39) If $\tan(\theta) = \frac{3}{4}$, what is $\sin(\theta)$? a) $\frac{5}{3}$ b) $\frac{4}{5}$ c) $\frac{3}{4}$ d) $\frac{3}{5}$

40) If x, y and z are the angles between 0 and $\frac{\pi}{2}$ such that $\tan x = \frac{1}{2}$, $\tan y = \frac{1}{5}$ and $\tan z = \frac{1}{8}$.

Find $x + y + z$

- a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$ e) None of these

41) A fair die is rolled. Probability of getting a number ≥ 4 :

- a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

42) A box has 5 red and 3 blue balls. One is chosen at random. Probability it is blue?

- a) $\frac{3}{5}$ b) $\frac{5}{8}$ c) $\frac{3}{8}$ d) $\frac{1}{3}$

43) A radioactive isotope has half-life 12 hours. If initial mass is 200 g, how much remains after 36 hours?

- A. 25 g B. 50 g C. 100 g D. 200 g

..... **HAPPY EASTER.....**